11.6: 10, 16, 26, 28, 30

10. $f(x,y,z) = \sqrt{x+yz} = (x+yz)^{1/2}$

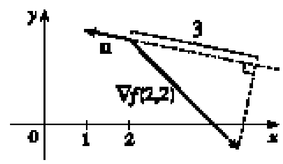
(a) $\nabla f(x,y,z) = \left\langle \frac{1}{2}(x+yz)^{-1/2}(1), \frac{1}{2}(x+yz)^{-1/2}(z), \frac{1}{2}(x+yz)^{-1/2}(y) \right\rangle$

$= \langle 1/(2\sqrt{x+yz}), z/(2\sqrt{x+yz}), y/(2\sqrt{x+yz}) \rangle$

(b) $\nabla f(1,3,1) = \left\langle \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle$

(c) $D_{\mathbf{u}} f(1,3,1) = \nabla f(1,3,1) \cdot \mathbf{u} = \left\langle \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle \cdot \left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right\rangle = \frac{2}{28} + \frac{3}{28} + \frac{18}{28} = \frac{23}{28}$.

16. $D_{\mathbf{u}} f(2,2) = \nabla f(2,2) \cdot \mathbf{u}$, the scalar projection of $\nabla f(2,2)$ onto $\mathbf{u}$, so we draw a perpendicular from the tip of $\nabla f(2,2)$ to the line containing $\mathbf{u}$. We can use the point $(2,2)$ to determine the scale of the axes, and we estimate the length of



the projection to be approximately 3.0 units. Since the angle between $\nabla f(2,2)$ and $\mathbf{u}$ is greater than 90°, the scalar projection is negative. Thus $D_{\mathbf{u}} f(2,2) \approx -3$.

26. The fisherman is traveling in the direction $\langle -80, -60 \rangle$. A unit vector in this direction is $\mathbf{u} = \frac{1}{100}\langle -80, -60 \rangle = \left\langle -\frac{4}{5}, -\frac{3}{5} \right\rangle$, and if the depth of the lake is given by $f(x,y) = 200 + 0.02x^2 - 0.001y^3$, then $\nabla f(x,y) = \langle 0.04x, -0.003y^2 \rangle$.

$D_{\mathbf{u}} f(80,60) = \nabla f(80,60) \cdot \mathbf{u} = \langle 3.2, -10.8 \rangle \cdot \left\langle -\frac{4}{5}, -\frac{3}{5} \right\rangle = 3.92$. Since $D_{\mathbf{u}} f(80,60)$ is positive, the depth of the lake is increasing near $(80,60)$ in the direction toward the buoy.

28. $\nabla T = -400 e^{-x^2-3y^2-9z^2} \langle x, 3y, 9z \rangle$

(a) $\mathbf{u} = \frac{1}{\sqrt{6}} \langle 1, -2, 1 \rangle$, $\nabla T(2,-1,2) = -400e^{-43} \langle 2, -3, 18 \rangle$ and

$$D_{\mathbf{u}} T(2,-1,2) = \left(-\frac{400e^{-43}}{\sqrt{6}} \right)(26) = -\frac{5200\sqrt{6}}{3e^{43}} \ {}^\circ\text{C/m}.$$

(b) $\nabla T(2,-1,2) = 400e^{-43} \langle -2, 3, -18 \rangle$ or equivalently $\langle -2, 3, -18 \rangle$.

(c) $|\nabla T| = 400 e^{-x^2-3y^2-9z^2} \sqrt{x^2 + 9y^2 + 8z^2}$ °C/m is the maximum rate of increase. At $(2,-1,2)$ the maximum rate of increase is $400e^{-43}\sqrt{337}$ °C/m.

30. $z = f(x,y) = 1000 - 0.005x^2 - 0.01y^2 \quad \Rightarrow \quad \nabla f(x,y) = \langle -0.01x, -0.02y \rangle$ and $\nabla f(60,40) = \langle -0.6, -0.8 \rangle$

(a) Due south is in the direction of the unit vector $\mathbf{u} = -\mathbf{j}$ and

$D_{\mathbf{u}} f(60,40) = \nabla f(60,40) \cdot \langle 0, -1 \rangle = \langle -0.6, -0.8 \rangle \cdot \langle 0, -1 \rangle = 0.8$. Thus, if you walk due south from $(60, 40, 966)$ you will ascend at a rate of 0.8 vertical meters per horizontal meter.

(b) Northwest is in the direction of the unit vector $\mathbf{u} = \frac{1}{\sqrt{2}} \langle -1, 1 \rangle$ and

$D_{\mathbf{u}} f(60,40) = \nabla f(60,40) \cdot \frac{1}{\sqrt{2}} \langle -1, 1 \rangle = \langle -0.6, -0.8 \rangle \cdot \frac{1}{\sqrt{2}} \langle -1, 1 \rangle = -\frac{0.2}{\sqrt{2}} \approx -0.14$. Thus, if you walk northwest from $(60, 40, 966)$ you will descend at a rate of approximately 0.14 vertical meters per horizontal meter.

(c) $\nabla f(60,40) = \langle -0.6, -0.8 \rangle$ is the direction of largest slope with a rate of ascent

$|\nabla f(60,40)| = \sqrt{(-0.6)^2 + (-0.8)^2} = 1$. The angle above the horizontal in which the path begins is given by

$\tan\theta = 1 \quad \Rightarrow \quad \theta = 45^\circ$.